

REPRESENTING FUNCTIONS

Big Picture

Functions are a way of relating one variable to another variable in order to find solutions for both variables. Graphs are a great way to visually represent functions and can often help the solution finding process.

Key Terms

Coordinate Plane: Used to represent sets of numbers.

Has two axes and an origin.

Origin: The point where two axes intersect.

Quadrant: Any one of four sections of the coordinate plane created by the two axes.

Coordinate: A pair of numbers that describe how far along the x -axis and y -axis a point is.

Function: A relationship between two variables (an input and an output) where there is exactly one output for each input.

Independent Variable: The input variable - a variable whose value does not depend on the value of another variable.

Dependent Variable: The output variable - a variable whose value depends on the value of an input variable.

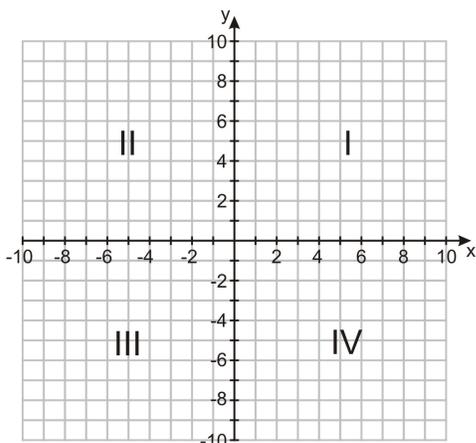
Domain: The set of values for the input variable of a function.

Range: The set of values for the output variable of a function.

Coordinate Plane

The **coordinate plane** can be used to represent pairs of numbers.

- Looks like two number lines that meet at right angles. These "number lines" are called axes (singular, axis).
 - The horizontal line is called the x -axis. The positive x -direction points to the right.
 - The vertical line is called the y -axis. The positive y -direction points upward.
- The **origin** is where the two axes intersect
- The axes split the coordinate plane into 4 **quadrants**
 - Moving counter-clockwise starting from the upper right quadrant, the quadrants are numbered I, II, III, and IV



Coordinates

A point on the coordinate plane is determined by its **coordinates** (x, y) .

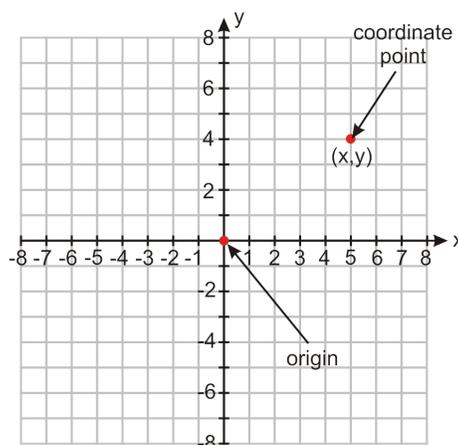
- The order that the coordinates are written in matters! Coordinates are always written with the x -coordinate first.
- The coordinates of the origin are $(0, 0)$.



The x -coordinate is also called the abscissa. The y -coordinate is also called the ordinate.

In the graph below, the coordinates of the point are $(5, 4)$.

- To reach the point from the origin, move 5 units to the *right* and 4 units *upward*.
- A negative x -coordinate means moving to the *left*, and a negative y -coordinate means moving *downward*.



Notes

REPRESENTING FUNCTIONS CONT.

Functions

Graphs are useful for showing relations and functions.

- A relation relates two or more variables (such as x and y).
 - An equation with two variables is an example of a relation. The two variables are related by performing the operations of the equation (multiplication, addition, etc.).
- A **function** is a type of relationship that relates one variable called an input to another variable called an output. The relation is such that there is *exactly* 1 output for every input.
- Typically x represents the input variable and y represents the output variable.
 - The input variable (x) is also called the **independent variable** because its value is not controlled by another variable.
 - The output variable (x) is the **dependent variable** because the output *depends* on the input variable.
- A function is often written as $f(x)$, which stands for *function of x* .
 - Examples: $f(x) = 2x - 5$, $f(x) = x^3$,
- Sometimes $f(x)$ is replaced with y .

Domain and Range

The **domain** and **range** describe the values that would make the function true.

- The domain is all the possible x -values (input values) of a function.
 - If the domain is all real numbers, the function is *continuous*. If the domain is a particular set of numbers (such as whole numbers only), the function is discrete.
- The range is all the possible y -values (output values) of a function.
 - The range of the function depends on the domain.

To find the domain of a function, determine which input values would make sense. Usually this means the input should produce a real number as the output. For example, if an x -value would make the denominator equal to 0, the output wouldn't be a real number.

• Example: $f(x) = \frac{1}{x - 2}$ Domain: $[x \neq 2]$

This means all real numbers *except* $x = 2$ are a part of the domain. If $x = 2$, the denominator would equal 0.

The easiest way to find the range of a function is to look at the graph and find where on the y -axis there are no x -values.

• Example: $f(x) = x^2$ Range: $[y \geq 0]$

This means all real numbers *greater than or equal* to 0 are a part of the range. x^2 is always positive, so it is not possible for the y -value to be less than 0.

Graphing Functions

A function can be graphed once the relation is known. The relation could be a rule or a table of values.

Example: Graph the function that has the following table of values.

Side of a square	0	1	2	3	4
Area of a square	0	1	4	9	16

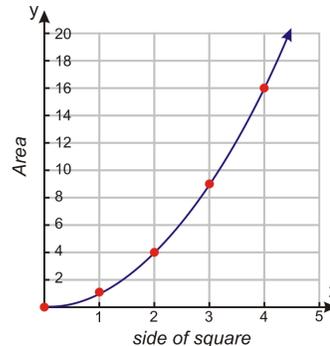
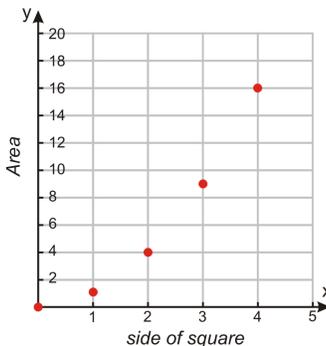
The graphs (to the right) only show quadrant I. It does not make sense to have a negative side or negative area, so they are not included in the domain and range.



When graphing functions, make sure to label the axes.

If drawing a graph when given a rule instead of a table of values, make a table of some values to plot.

- Usually picking integer values is best because it makes calculations easier.
- Connect the points if the graph is continuous.



Vertical Line Test

When graphing a relation, the vertical line test can be used to see if the relation is a function.

- If you can draw a vertical line that crosses the graph in more than one place, the relation is *not* a function.

