Dimensional Analysis

Andrew Gloag
Anne Gloag
Melissa Kramer

AUTHORS
Andrew Gloag
Anne Gloag
Melissa Kramer

EDITORS
Annamaria Farbizio

To access a customizable version of this book, as well as other interactive content, visit www.ck12.org

CK-12 Foundation is a non-profit organization with a mission to reduce the cost of textbook materials for the K-12 market both in the U.S. and worldwide. Using an open-content, web-based collaborative model termed the **FlexBook®**, CK-12 intends to pioneer the generation and distribution of high-quality educational content that will serve both as core text as well as provide an adaptive environment for learning, powered through the **FlexBook Platform®**.

Printed: July 8, 2012

flexbook
next generation textbooks

CONCEPT 1 Dimensional Analysis

Here you'll learn how to convert or cancel dimensions in order to solve real-world problems.

What if you went to the grocery store and bought 3 gallons of milk? Could you determine how many pints of milk you purchased? Or how about if you bought 16 pints of milk? How many gallons would this be? In this Concept, you'll learn to make conversions like these so that you can solve real-world problems.

Guidance

Real-world information is given in **dimensions**, or the units in which the value is measured. For example, the following are all examples of dimensions.

- Inches
- Feet
- Liters
- Micrograms
- Acres
- Hours
- Pounds
- Students

Analyzing dimensions can help you solve problems in travel, astronomy, physics, engineering, forensics, and quality. Solving problems by converting dimensions or canceling dimensions is the focus of this Concept.

Consider the distance formula $distance = rate \cdot time$. This formula can be rewritten for *rate*. $rate = \frac{distance}{time}$. If *distance* is measured in kilometers, and *time* is measured in hours, the rate would have the dimensions $\frac{kilometers}{hours}$.

You can treat dimensions as variables. Identical units can divide out, or cancel. For example, $\frac{kilometers}{hour} \cdot hour \rightarrow \frac{kilometers}{\cancel{hour}} \cdot \cancel{hour} \rightarrow kilometers$.

Sometimes the units will not divide out. In this case, a **conversion factor** is needed.

Example A *Convert* $\frac{35\ kilometers}{hour}$ *to meters.*

Solution:

Since *kilometers* $\neq$*meters*, you need to **convert** kilometers to meters to get the answer. You know there are 1,000 meters in a kilometer. Therefore, you will need to multiply the original dimension by this factor.

$$\frac{35\ kilometers}{hour} \cdot \frac{1000\ meters}{1\ kilometer} \rightarrow \frac{35\ \cancel{kilometers}}{hour} \cdot \frac{1000\ meters}{1\ \cancel{kilometer}} = \frac{35(1000) meters}{hour}.$$
$$\frac{35\ kilometers}{hour} = \frac{35,000\ meters}{hour}$$

The process of using units or dimensions to help solve a problem is called **dimensional analysis**. It is very useful in chemistry and travel, as shown in the examples below.

Example B *How many seconds are in a month?*

Solution: This situation can be solved easily using multiplication. However, the process you use when multiplying the values together is an example of dimensional analysis.

Begin with what you know:

- 60 seconds in one minute
- 60 minutes in one hour
- 24 hours in one day
- Approximately 30 days in one month

Now write the expression to convert the seconds in one minute to one month.

$$\frac{60\ seconds}{1\ minute} \cdot \frac{60\ minutes}{1\ hour} \cdot \frac{24\ hours}{1\ day} \cdot \frac{30\ days}{1\ month}$$

Identical units cross-cancel.

$$\frac{60\ seconds}{1\ \cancel{minute}} \cdot \frac{60\ \cancel{minutes}}{1\ \cancel{hour}} \cdot \frac{24\ \cancel{hours}}{1\ \cancel{day}} \cdot \frac{30\ \cancel{days}}{1\ month}$$

Multiply the fractions together.

$$\frac{60 \cdot 60 \cdot 24 \cdot 30\ seconds}{1 \cdot 1 \cdot 1 \cdot 1\ month} = 2,592,000 \frac{seconds}{month}$$

Example C *How many grams are in 5 pounds?*

Solution: Begin by writing all the conversions you know related to this situation.

$$1\ gram \approx 0.0353\ ounces$$
$$16\ ounces = 1\ pound$$

Write your dimensional analysis.

$$5\ pounds \cdot \frac{16\ ounces}{1\ pound} \cdot \frac{1\ gram}{0.0353\ ounce}$$

Cross-cancel identical units and multiply.

$$5\ \cancel{pounds} \cdot \frac{16\ \cancel{ounces}}{1\ \cancel{pound}} \cdot \frac{1\ gram}{0.0353\ \cancel{ounce}} = 2226.29\ grams$$

A long list of conversion factors can be found at this website .

Vocabulary

Dimensional analysis: The process of using units or dimensions to help solve a problem is called ***dimensional analysis***. It is very useful in chemistry and travel.

Guided Practice

You are traveling in Europe and want to know how fast to drive to maximize fuel efficiency. The optimal driving speed for fuel efficiency is 55 miles per hour. How fast would that be in kilometers per hour?

Solution:

Since 1 mile is approximately 1.6 kilometers:

$\frac{55\ miles}{hour} \cdot \frac{1.6\ kilometers}{1\ mile} \rightarrow \frac{55\ \cancel{miles}}{hour} \cdot \frac{1.6\ kilometers}{1\ \cancel{mile}} \rightarrow \frac{88\ kilometers}{hour}$

The optimal speed for fuel efficiency is 88 kilometers per hour.

Practice

1. *True or false?* Dimensional analysis is the study of space and time.
2. By using dimensional analysis, what happens to identical units that appear diagonally in the multiplication of fractions?
3. How many feet are in a mile?
4. How many inches are in a mile?
5. How many seconds are in a day?
6. How many seconds are in a year?
7. How many feet are in a furlong?
8. How many inches are in 100 yards (one football field)?
9. How many centimeters are in 5 inches?
10. How many meters are between first and second base (90 feet)?

11. How many meters are in 16 yards?
12. How many cups are in 6 liters?
13. How many cubic inches make up one ounce?
14. How many milliliters make up 8 ounces?
15. How many grams are in 100 pounds?
16. An allergy pill contains 25 mg of Diphenhydramine. If $1\ gram = 15.432\ grains$, how many grains of this medication are in the allergy pill?
17. A healthy individual's heart beats about 68 times per minute. How many beats per hour is this?
18. You live 6.2 miles from the grocery store. How many fathoms is this? ($6\ feet = 1\ fathom$)
19. The cost of gas in England is 96.4 *pound sterling/liter*. How much is this in *U.S. dollars/gallon*? ($3.875\ litres = 1\ gallon$ and $1.47\ US\$ = 1\ pound\ sterling$)
20. Light travels $\frac{186{,}000\ miles}{second}$. How long is one light year?
21. Another way to describe light years is in *astronomical units*. If $1\ light\ year = 63,240\ AU(astronomical\ units)$, how far in AUs is Alpha Centauri, which is 4.32 light years from the Earth?
22. How many square feet is 16 acres?
23. A person weighs 264 pounds. How many kilograms is this weight?
24. A car is traveling 65 *miles/hour* and crosses into Canada. What is this speed in *km/hr*?
25. A large soda cup holds 32 ounces. What is this capacity in cubic inches?
26. A space shuttle travels 28,000 mph. What is this distance in *feet/second*?
27. How many hours are in a fortnight (two weeks)?
28. How many fortnights (two-week periods) are in 2 years?
29. A semi truck weighs 32,000 pounds empty. How many tons is this weight?
30. Which has the greatest volume: a 2-liter bottle of soda, one gallon of water, or 10 pints of human blood?

Mixed Review

31. Solve for $x : -2x+8 = 8(1-4x)$.
32. Simplify: $3-2(5-8h)+13h \cdot 3$.
33. Find the difference: $-26.375 - \left(-14\frac{1}{8}\right)$.
34. Find the product: $-2\frac{3}{7} \cdot \frac{9}{10}$.
35. Simplify: $\sqrt{80}$.
36. Is $5.\bar{5}$ an irrational number? Explain your answer.

Use the relation given for the following questions: $\left\{(0,8),(1,4),(2,2),(3,1),\left(4,\frac{1}{2}\right),\left(5,\frac{1}{4}\right)\right\}$.

37. State the domain.
38. State the range.
39. Is this relation a function? Explain your answer.
40. What seems to be the pattern in this relation?